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Temperature and Velocity Profiles along a Vertical Hot Plate in a Compressible Fluid Considering the Effect of Buoyancy

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Nomenclature

T	= absolute temperature
c_p	= specific heat at constant temperature
c_v	= specific heat at constant volume
k	= coefficient of heat conductivity
γ	= ratio of the specific heats
P_r	= Prandtl's number
R/J	= $c_p - c_v$
J	= joule equivalent
g	= acceleration due to gravity
β	= coefficient of cubical expansion
ρ	= density

Subscripts

ω	= conditions at the wall
0	= conditions at $n = 0$
α	= outside the boundary layer

Introduction

THE original Blasius solution of the equations of the boundary-layer flow over a plate has been extended in various ways by different authors For instance, Chapman and Rubesin[1] have considered the flow and heat transfer in the boundary layer of a compressible fluid with zero pressure gradient over a plate, taking viscous dissipation into consideration, with certain assumptions regarding c_p,μ and the Prandtl's number P_r Pohlhausen,[2] on the other hand, solved the equation of liquid boundary-layer flow over a vertical hot plate taking buoyancy into consideration, but neglecting the frictional heat This problem was experimentally studied by Schmidt and Beckmann,[4] and Pohlhausen's theoretical results agreed fairly well with the experimental results The object of the present note is to show that the problem of the forementioned authors can be solved for a compressible flow under zero pressure gradient ignoring frictional heat, but taking buoyancy into consideration The Mises transformation[3] is found to be effective here as in the Karman-Tsien method

Consider a boundary layer in contact with a vertical hot plate and take the x axis vertically upward along the hot plate The motion is steady and is supposed to be caused by the difference between the weight and the buoyancy in the gravitational field of the earth The small pressure gradient being neglected, the equations of motion in the boundary layer are

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) + \rho g \beta (T - T_\infty) \tag{1}$$

$$\rho u \frac{\partial}{\partial x}(c_p T) + \rho v \frac{\partial}{\partial y}(c_p T) = \frac{\partial}{\partial y}\left(K \frac{\partial T}{\partial y}\right) \tag{2}$$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{3}$$

Frictional heat also has been neglected We shall solve the problem under the following assumptions:

1) $c_p = \text{const}$
2) $P_r = c_p\mu/k =$ Prandtl's number
3) $\mu/\mu_\infty = cT/T_\infty$ when $c = \text{const}$

On putting $\theta = (T - T_\infty)/(T_\infty - T_\infty)$, T_ω being the temperature on the wall, the equations are reduced to the form

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) + \rho a \theta \tag{4}$$

$$\rho u \frac{\partial \theta}{\partial x} + \rho v \frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial}{\partial y}\left(\mu \frac{\partial \theta}{\partial y}\right) \tag{5}$$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{6}$$

when

$$a = g\beta(T_\omega - T_\infty)$$

We introduce a stream function ψ by the equations

$$u = \frac{\rho_\infty}{\rho}\frac{\partial \psi}{\partial y} \qquad v = -\frac{\rho_\infty}{\rho}\frac{\partial \psi}{\partial x} \tag{7}$$

Now effect a Mises transformation $(x,y) \to (x,\psi)$ according to

$$\left(\frac{\partial}{\partial y}\right) = \left(\frac{\rho u}{\rho_\infty}\right)\frac{\partial}{\partial \psi}$$

$$\left(\frac{\partial}{\partial_n}\right) = \left(\frac{\partial}{\partial x}\right)_\psi - \left(\frac{\rho v}{\rho_\infty}\right)\frac{\partial}{\partial \psi}$$

when Eqs (4) and (5) reduce to

$$u\frac{\partial u}{\partial x} = \frac{\mu_\infty}{\rho_\infty} u \frac{\partial}{\partial \psi}\left(cu\frac{\partial u}{\partial \psi}\right) + a\theta \tag{8}$$

$$\frac{\partial \theta}{\partial x} = \frac{\mu_\infty}{P\rho_\infty}\frac{\partial}{\partial \psi}\left(cu\frac{\partial v}{\partial \psi}\right) \tag{9}$$

We introduce dimensionless variables in the usual way by the substitutions $x^* = x/L$, $u^* = u/U$, $\mu^* = \mu/\mu_\infty$, $\psi^* =$

Received September 4, 1963 I take this opportunity to express my gratitude to R N Bhattacharyya of the Mathematics Department, Jadavpur University, for his kind help and guidance in the preparation of this paper

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$\psi/(\nu_\infty ULc)^{1/2}$, and $\rho^* = \rho/\rho_\infty$ for which (8) and (9) become

$$u^* \frac{\partial u^*}{\partial x^*} = u^* \frac{\partial}{\partial \psi^*}\left(u^* \frac{\partial \mu^*}{\partial \psi^*}\right) + b\theta \tag{10}$$

$$\frac{\partial \theta}{\partial \psi_*} = \frac{1}{P}\frac{\partial}{\partial \psi^*}\left(u^* \frac{\partial \theta}{\partial \psi^*}\right) \tag{11}$$

when $b = aL/U^2$ (L being as yet arbitrary)

Let us assume a set of solutions of the form

$$u^* = Ax^{*p}f'(\eta) \qquad f(\eta) = \frac{\psi^*}{x^*q} \qquad \theta = Bx^{*\gamma}\xi(\eta)$$

Equations (10) and (11) become

$$A^2x^{*2p-1}[pf'^2 - qff''] = A^3x^{*3p-2q}f''' + Bbx^{*\gamma}\xi \tag{12}$$

$$Bx^{*\gamma-1}[\gamma f'\xi - q\xi' f] = \frac{AB}{P\gamma} x^{*p-2q+\gamma}\xi'' \tag{13}$$

If we now assume $p = \frac{1}{2}, \gamma = 0, q = \frac{3}{4}, A = \frac{1}{4}, B = 1, b = A^3$, giving $L = \frac{1}{64}(u^2/a)$, then Eqs (12) and (13) take the form

$$f''' + 3ff'' - 2f'^2 + \xi = 0 \tag{14}$$

$$\xi'' + 3P_r f\xi' = 0 \tag{15}$$

The boundary conditions now become, $f = f' = 0$, $\xi = 1$ for $\eta = 0$, and $f' = 0$, $\xi = 0$, $\eta = \infty$ Equations (14) and (15) are exactly identical with those deduced by Pohlhausen when he discussed the flow of a liquid along a hot vertical plate Taking buoyancy forces into consideration, the curves showing the solution already have been given by the forementioned author for different values of P Thus it is shown that the solution of the problem of compressible flow may be reduced to that for the problem of incompressible flow which now takes the form $n^* = \frac{1}{4}n^{*1/2}f'(\eta)$ $f(\eta) = \psi^* n^{*3/4}$, $\theta = \xi(\eta)$

Heat Transfer

The quantity of heat transfer per unit time and area is

$$q(x) = -k\left(\frac{\partial T}{\partial y}\right) = -\frac{k\rho u}{\rho_\infty}\left(\frac{\partial T}{\partial \psi}\right) = -\frac{k\rho u(T_\omega - T_\infty)}{\rho_\infty(\nu_\infty ULC)^{1/2}}\frac{\partial \theta}{\partial \psi^*}$$

$$= -\frac{k\rho u^* U(T_\omega - T_\infty)}{\rho_\infty(\nu_\infty ULC)^{1/2}}\frac{1}{f'(\eta)x^{3/4}}\left(\frac{\partial \theta}{\partial \eta}\right)$$

$$= -\frac{k\rho U(T_\omega - T_\infty)}{4\rho_\infty(\nu_\infty ULC)^{1/2}}\left(\frac{\partial \theta}{\partial \eta}\right)x^{*-1/4}$$

The quantity of heat transfer per unit time and area from the plate to the fluid at a distance x

$$= -\frac{k_0\rho_0(T_\omega - T_\infty)}{4\rho_\infty(\nu_\infty ULC)^{1/2}}UL^{1/4}\left(\frac{\partial \theta}{\partial \eta}\right)_0 x^{-1/4}$$

$$= -\frac{k_0}{4}\frac{(T_\omega - T_\infty)}{(\nu_0)^{1/2}}\frac{U^{1/2}}{L^{1/4}}\left(\frac{\partial \theta}{\partial \eta}\right)_0 x^{-1/4}$$

$$\left[\frac{\mu}{\mu_\infty} = C\frac{T}{T_\infty} = C\frac{\rho_\infty}{\rho};\ \text{therefore}\left(\frac{\nu}{\nu_\infty}\right)^{1/2} = c^{1/2}\frac{\rho_\infty}{\rho}\right]$$

Total heat transfer by a plate of length l and width b is

$$Q = b\int_0^L a(x)dx = -\frac{bk_0}{3}(T_\omega - T_\infty)\left(\frac{\partial \theta}{\partial \eta}\right)_0\left(\frac{UL}{\nu_0}\right)^{1/2}$$

$$= -\frac{bk_0}{3}(T_\omega - T_\infty)\left(\frac{\partial \theta}{\partial \eta}\right)_0 R_0^{1/2} \tag{16}$$

where $(\partial\theta/\partial\eta)_0$ depends upon Prandtl's number The mean Nusselt number is defined by

$$Q = bk_0(T_\omega - T_\infty)Nm$$
$$Nm = -\frac{1}{3}\left(\frac{\partial \theta}{\partial \eta}\right)_0 R_0^{1/2} \tag{17}$$

It should be noted that Q as well as the Nusselt number depends upon the Reynolds number and Prandtl number as usual

$$\tau(x) = \mu\left(\frac{\partial u}{\partial y}\right) = \frac{\mu\rho u}{\rho_\infty}\left(\frac{\partial u}{\partial \psi}\right) = \frac{\mu\rho u^* L^2}{\rho_\infty(\nu_\infty ULC)^{1/2}}\frac{1}{f'(\eta)x^{*3/4}}\left(\frac{\partial u^*}{\partial \eta}\right)$$

Local skin friction is $\tau_0(x) = [(\mu_0\rho_0 U^3)^{1/2}/16L^{3/4}]f''(0)x^{1/4}$ The total value of skin friction over a portion of the plate of width b and length L is D_f:

$$D_f = b\int_0^L \tau_0(x)dx = \frac{b(\rho_0\mu_0U^3)^{1/2}}{20}L^{1/2}f'(0) \tag{18}$$

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Exact First-Order Navigation-Guidance Mechanization and Error Propagation Equations for Two-Body Reference Orbits

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Nomenclature

r	=	radial distance to dynamical center
p	=	semilatus rectum
n^*	=	$(\mu/p^3)^{1/2}$ = modified mean motion parameter
v	=	true anomaly
e	=	eccentricity
τ	=	$t - t_0$ = time since initial epoch
Δx	=	inertial horizontal in-plane position error
Δy	=	inertial out-of-plane position error

Received August 7, 1963; revision received December 13, 1963

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